Solutions

Math 2D Quiz 1 Afternoon - March 31, 2016

Please put ID on back for redistribution!

Show all of your work. *There is a question on the back side.

1. [10pts] Compute the following integral:

$$\int_0^1 \int_0^1 (x+y)^2 \, dy \, dx.$$

Simplify your answer to a single fraction.

$$\to \int_0^1 \frac{(x+y)^3}{3} \Big|_{y=0}^{y=1} dx = \int_0^1 \frac{(x+1)^3 - x^3}{3} dx \quad +5$$

$$= \frac{(x+1)^4 - x^4}{12} \Big|_{x=0}^{x=1} = \frac{(2^4 - 1^4) - (1^4 - 0)}{12} = \frac{16-1-1}{12} = \boxed{\frac{7}{6}} \quad +5$$

If used Fubini, it's no different,

$$\int_0^1 \int_0^1 (x+y)^2 \, dx \, dy = \int_0^1 \frac{(x+y)^3}{3} \Big|_{x=0}^{1} dy$$

$$= \int_0^1 \frac{(y+1)^3 - y^3}{3} dy \quad (+5) \quad = \frac{(y+1)^4 - y^4}{12} \Big|_{y=0}^{y=1}$$

$$= \frac{(2^4 - 1^4) - (1^4 - 0)}{12} = \frac{16-1-1}{12} = \boxed{\frac{7}{6}} \quad (+5)$$

2. [10pts] Find the volume of the solid that lies under the plane $4x + 6y - 2z + 14 = 0$ and above the rectangle $R = \{(x, y) : -1 \le x \le 2, -1 \le y \le 1\}$. Simplify your answer to a single number.

First, $4x+6y-2z+14 \Rightarrow 2z = 4x+6y+14$,

+2 $\boxed{z = 2x+3y+7}$, "height fn"

So, $V = \iint_R (2x+3y+7)\,dA = \int_{-1}^{2}\int_{-1}^{1} (2x+3y+7)\,dy\,dx$

$= \int_{-1}^{2} 2xy + \frac{3y^2}{2} + 7y \Big|_{y=-1}^{y=1} dx$ ☆ utilize symmetry!

+5

$= \int_{-1}^{2} (4x+14)\,dx = 2x^2+14x \Big|_{-1}^{2}$

$= 8+28-(2-14) = \boxed{48}$ +3

If $dx\,dy$: $\int_{-1}^{+1}\int_{-1}^{2} (2x+3y+7)\,dx\,dy = \int_{-1}^{1} x^2+3xy+7x \Big|_{x=-1}^{x=2} dy$

$= \int_{-1}^{1} \left[(\overbrace{4+14}^{18}+6y) - (\overbrace{1-3y-7}^{-6}) \right] dy$

(+5)

$= \int_{-1}^{1} (9y+24)\,dy$ ☆ Again, utilize symmetry!

$= \frac{9y^2}{2} + 24y \Big|_{-1}^{1} = \boxed{48}$ (+3)